

THE TELLING DILEMMA: TYPES OF MATHEMATICAL TELLING IN INQUIRY

Brandon K. Singleton
University of Wisconsin-Madison
bksingleton@wisc.edu

Teacher telling continues to be poorly understood within inquiry. In this paper I extend prior efforts to reimagine telling within contemporary pedagogical thought. Using a case study, I investigated a well-regarded teacher's use of mathematical telling while supporting groups and individuals working on tasks. The teacher used seven unique types of mathematical telling: assess, interpret, qualify, clarify task, guide, disclose, and validate. This mathematical telling framework aids in the identification of subtle telling, the recognition of implicit telling, and the acknowledgment of explicit telling. Telling practices should be conceptualized and evaluated contextually.

Keywords: Classroom Discourse; Instructional Activities and Practices

Introduction

At the heart of mathematics teaching lies an enduring dilemma of how and when to tell. This dilemma surfaces within a broader struggle to privilege the spontaneous thought and activity of the child while simultaneously cultivating and enculturating the child into a scientific and socialized society. Contemporary pedagogical practice is driven by progressive reforms advocating student-driven inquiry, on the one hand, and increased standardization and accountability to prescribed conventional knowledge of mathematics, on the other. These demands require a give-and-take approach since students cannot just independently discover everything they are expected to know, nor do they simply absorb foreign ideas that are explained to them. To teach, then, is to skillfully manage the intersection of the child and the curriculum. The telling dilemma repeatedly surfaces in that work. Teachers grapple with how to manage learning through tasks (Henningsen & Stein, 1997; Simon, 1995), how and when to steer class discussions (Ball, 1993; Chazan & Ball, 1999), how to reorient student inquiry that is misguided or trivial (Ball, 1993), and how to help diverse students access and benefit from implicit forms of knowing and learning in classroom discourse practices (Lubienski, 2002).

The issue is not whether always to tell or never to tell, but rather when and how to tell (Baxter & Williams, 2010; Chazan & Ball, 1999; Lobato, Clarke, & Ellis, 2005). Nevertheless, explicit teacher telling of mathematical *content* remains taboo as teacher practice is framed through less mathematically obtrusive constructs. Teaching has been framed in terms of orchestrating discourse (Lampert, 1990; Rittenhouse, 1998; Staples, 2007; Stein, Engle, Smith, & Hughes, 2008; Wood, 1998), facilitating collaboration and interaction (Clarke, 1997; Cohen, 1994; Dekker & Elshout-Mohr, 2004; Webb, 2009; Webb et al., 2009), and posing high-quality tasks (Henningsen & Stein, 1997; Simon & Tzur, 2004). When research does acknowledge and discuss teacher moves related to telling, either these telling moves are believed to compromise the quality of learning (Chiu, 2004; Dekker & Elshout-Mohr, 2004; Tyminski, 2010), or the mathematical substance originating from the teacher is absent or downplayed (e.g., Chazan & Ball, 1999; Smith, 1996; Staples, 2007).

Arguably, such treatment of teacher telling is overly conservative given that the teacher's mathematical discourse can be a tool for promoting student sense making (Baxter & Williams, 2010; Ding, Li, Piccolo, & Kulm, 2007; Lobato et al., 2005). Further understanding is needed of how teachers tell or speak assertively with students about mathematics, especially while they grapple with the major challenges of teaching within inquiry.

Although teacher telling might occur in various instructional formats, this study focused on telling during teacher interventions with collaborative groups. In the whole class setting, teachers can

rely on student contributions of content to advance class discussions (e.g., Stein et al., 2008) whereas beforehand in the collaborative group setting, teachers must prepare students so they are able to make those contributions and access the contributions of others. During students' struggles with tasks, teachers may encounter opportunities to engage mathematically with particular students in response to specific needs. The research question of this study was, "What types of mathematical telling does a well-regarded inquiry-based teacher use to support students while intervening with small groups and individuals working on mathematical tasks?"

Framework

To address my research question I briefly discuss inquiry before framing telling more rigorously. The term "inquiry" in this study is meant as a broad descriptor of instruction that aspires to privilege student thinking and activity during the cultivation of curricular standards. Without claiming that inquiry is a well-defined and homogenous mode of instruction, I argue that the teacher in this case study upheld the basic aspiration of inquiry as she enacted various practices. The teacher varied her instructional format between whole-class discussion, small-group work, and individual work. She introduced non-routine mathematical tasks and monitored student progress. She facilitated students in sharing and critiquing one another's reasoning. The teacher helped students form complete and correct justifications of their ideas. She gave detailed feedback on their work in and out of class. These practices are believed to support student inquiry into mathematics (see Clarke, 1997; Staples, 2007; Stein et al., 2008; Webb et al., 2009).

To study mathematical telling, I followed Lobato et al. (2005) in framing telling as a phenomenon of verbal discourse with three main attributes: form, content, and function. These three attributes are discussed below.

The form of discourse refers to its organization and grammatical structure. Discourse is produced into questions, statements, requests, commands, and so on, by selecting from a complex variety of verb tenses and grammatical syntax. Although an utterance derives meaning in part from its grammatical form, the form does not uniquely determine whether that utterance is telling. Questions can tell and statements can question (Lobato et al., 2005, p. 9).

The content of discourse refers to the objects and ideas denoted by particular words and phrases. Although language does not literally transmit fixed meanings from speaker to listener (von Glasersfeld, 1995), the references of an utterance are important indexical markers for objects and their negotiated meanings. A mathematical reference is an indicator of telling even if the internal mental referent of the speaker's utterance is inaccessible to the listener.

The function of discourse refers to its purpose in a situated activity. Function is simultaneously determined by a speaker's intentions, a listener's interpretations, and the nature of the activity itself that is enacted through the discourse (Lobato et al., 2005). Although these three elements may or may not align with one another and are often difficult to infer, I managed this complexity by limiting my research focus to the verbalized discourse of the lesson. My inferences were therefore centered on the observable function of the discourse during an unraveling event rather than on the layers of meanings for the participants.

In coordinating these three attributes of discourse, I did not use form directly, but I was sensitive to form while making sense of content and function to overcome unwarranted prejudices about telling based on form (e.g., "questions don't tell"). I defined mathematical telling, then, as discourse that contained mathematical *content* (indexical references) and served the *function* of inserting something new mathematically into the conversation. During later analysis the "something new" came to mean the insertion of mathematical ideas, structure, constraints, or acts that were not in play before the utterance.

Method

I conducted a case study of a well-regarded teacher educator teaching one class at a large four-year private university. The course was a mathematics content course (the second of a two-course series) designed for pre-service elementary teachers and included topics such as fractional reasoning, probability, and statistics. The class met twice weekly for 110 minutes per class. I observed and analyzed two full curricular units of instruction, each with seven lessons. I typed field notes to index the events of each class and recorded conversations in abbreviated form. All classroom activities were filmed, and subsequently all interactions between the teacher and students during task-work were transcribed. These transcriptions formed the core data for analysis, supplemented by my field notes and the task-sheets and handouts from the course.

I first coded for mathematical telling using the definition articulated earlier. The unit size for a mathematical telling act was generally a teacher turn. I then started differentiating mathematical telling acts from one another by attending to their mathematical content and the relationship of that content to the students' inquiry. I analyzed small subsets of data and generated provisional categories of telling. I then analyzed my set of categories, working to find general overlaps, inconsistencies, ambiguities, and so on. Changes to the categories were then taken back into the data. I continued this process until a provisional set of codes had surfaced. I briefly explained the codes to an informed peer who coded a small portion of data. The questions and discrepancies from this exercise helped me to modify, invent, dissolve, and reorganize codes and improve their written theoretical definitions. I also picked two fresh lesson transcripts to code twice, with a one-week interval between coding. After comparing my first and second reading of each transcript, I identified potential technical overlaps at the boundaries of codes and updated code definitions to set precedents for how to handle similar cases. I had used about half of my data to solidify this coding scheme, and I coded the remaining half comfortably.

Results

There were seven telling types in the eventual framework. As I interpreted the results, two telling types (*guide* and *disclose*) contained significant internal variation that I sorted into four purposes each in order to understand them better. For presentation, I ordered the seven types roughly from the least obtrusive to the most obtrusive.

Assess

The first type of mathematical telling is to assess. Not every assessment is telling (such as "How did you solve the problem?"), so a telling assessment is a mathematically structured or constrained request beyond just an open elicitation of student thinking. Even though a telling assessment does not reveal the answer or solution, it imposes a new idea or constraint and is therefore a form of telling. A responding student shares not only his or her own thinking but shapes it in order to either conform or object to the constraint embedded in the assessment.

A typical example occurred frequently during a particular lesson. The teacher would assess students' understanding of two fraction images (i.e. conceptual representations) by asking of their reasoning, "Is that partitioning or iterating?" Despite seeming innocuous, there is considerable mathematical structure embedded in such discourse. The assessment restricted students to those two images despite there being reasonable alternatives that at least one student was observed to use. Furthermore, it created a forced choice between the images even though many students at times believed they were appealing to both images within a single justification. Other assessments were even more complex when they implied potential connections (e.g., between fractions and the operation of division) that students had not yet considered.

Interpret

The second type of mathematical telling is to interpret students' mathematical formulations. The teacher clarified or characterized a student utterance by rephrasing, summarizing, generalizing, condensing, or inferring unspoken pieces of a student thought. Even though the teacher attributed such interpretations to students, they were nevertheless filtered through the teacher's own conceptual grid of meanings and brought something new into play.

Qualify

The third type of mathematical telling is to qualify the mathematics. The teacher qualified a mathematical part of the conversation according to human experience such as student feelings, motivation, or common sense. Because some students saw mathematics as threatening, irrelevant, contrived, difficult, or tedious, the teacher attended to these issues. For example, the teacher downplayed student errors to minimize embarrassment, attached value to the activity or task, acknowledged interesting contributions, and characterized (valid) justifications as awkward or natural. The discourse in these telling acts mediated the students' relationships with the mathematics and consequentially communicated something new about mathematics. This telling type was a surprising result and suggests that how individuals think and feel *about* mathematics is inextricably related to what mathematics *is* to them.

As a brief example, a student worked on a probability question about drawing colored balls repeatedly from a bag. The students had not yet developed a formula or procedure and were calculating the probabilities by making lists of possible outcomes and determining the fractional part of the outcomes that answered the question. One student began to find the activity tedious:

Student: So like, what's the point of listing all the combinations? What's it teaching us?

Teacher: What's this teaching you? Well for a lot of— (*not coded mathematical telling*)

Student: Like to be patient.

Teacher: No, no, no [laughs]. It's not teaching you to be patient. It's first of all helping us think about the situation, like what's really involved here in the situation. We have these shortcuts. We can't make meaning of the shortcuts if we don't know what's actually going on. And so I know you have these nice shortcuts like multiplication, but if the shortcuts don't make sense in terms of the situation then they are meaningless. But writing out the combinations can help us see where the shortcuts are going to come in. (*qualify*)

Here the teacher qualified the value of the task by affirming that doing mathematics is about making meaning of situations rather than executing shortcuts.

Clarify Task

The fourth type of mathematical telling is to clarify the task. The teacher clarified the mathematics of the task, question, or activity without addressing the actual solution. Two main purposes for clarifying the task were to provide basic instructions for student engagement and to clarify the mathematical meanings prerequisite to engaging in the task.

Guide

The fifth type of mathematical telling is to guide. The teacher guided students while they developed solutions, constructed justifications, discussed concepts and addressed errors. The key characteristic of teacher guidance was that the mathematical substance it contributed remained partially incomplete or unresolved, requiring the student to act on, complete, or incorporate it into his or her work. The teacher guided students for four main purposes (see Table 1): focus toward or away from an idea, lead students into productive ways of thinking, address reasoning errors, and give helpful hints and suggestions.

Table 1: Purposes of Guidance

Focus	Direct student attention, encourage or discourage student approaches, point to parallel examples or previous experiences
Lead	Pose questions or next steps, structure student justifications or explanations, ask rhetorical questions, help students draw a conclusion or inference
Address reasoning errors	Identify a student contradiction, invalid assumption, or deficiency; give counter-arguments; workshop a student justification; locate or correct an error
Give hints, suggestions	Suggest a method, bound or estimate the solution, interpret the problem, state principles or guidelines, explicate the criteria for an acceptable solution

In the example that follows, a student was solving the following question with Cuisenaire rods: “If purple is 2, what is the value of black?” She had placed a purple rod adjacent to a black rod, and then lined up seven small white rods alongside them as shown in Figure 1. The student first expressed confusion with the problem, using language such as, “The purple is four out of seven, the black is seven out of seven.” The teacher guided the student by first *focusing* her attention toward the given information in the problem (that the purple rod has length 2), but the student could not reconcile this given information with her propensity to think of the little white blocks as units or ones (she continually referred to purple as “four”). The teacher *addressed the reasoning error* by explicitly inviting the student to reconcile her naming of purple as four and the problem’s given assumption that purple is two. When the student was unable to do so, the teacher followed up with a series of crucial *leading* moves (presented below) that marked the climax of the interaction and enabled the student to subsequently reason her way to the solution:

Purple						
Black						
W	W	W	W	W	W	W

Figure 1. Student’s arrangement of purple, black and white Cuisenaire rods.

Teacher: So if this [purple] is, if this is two, what’s this? [picks up one white block] (*Guide-Lead*)

Student: A half.

Teacher: Okay what, how did you get that? Why do you know it’s a half? (*Not mathematical telling*)

Student: Because one half multiplied by four is two.

Teacher: Okay, so what would one be? (*Guide-Lead*)

Student: One would be, [shows the amount of two whites with fingers]

Teacher: Yeah. Okay, so now use this to figure out what black is. So you know that these [whites] are not one. (*Guide-Lead*)

Soon after this, the student correctly identified black as three and a half. The teacher’s leading questions were not just non-mathematical process help (see Dekker & Elshout-Mohr, 2004), nor were they “funneling” questions that trivialized the mathematical concept for the student (Wood, 1998). The first leading question firmly established the given assumption that purple is two as the premise from which to make a new deduction about the quantity that one white represents. This information helped the student to stop thinking of purple as four and white as one, and was the pivotal turning point after which the student began to reason appropriately about the situation. Guidance such as this was a frequent tool for this teacher and is a powerful form of telling whose mathematical substance should not be downplayed.

Disclose

The sixth type of mathematical telling is to disclose. A disclosure, unlike guidance, was more mathematically complete and resolved and usually revealed a solution component, an alternative solution, an explanation, a justification, a norm, or a convention. The teacher disclosed this information while discussing complex concepts, answering student questions, helping students construct solutions and justifications, and refining student work. Four main purposes of disclosure emerged (see Table 2): amplify student input, explain mathematical concepts, model appropriate reasoning, and provide norms and expectations.

Table 2: Purposes of Disclosure

Amplify student input	Elaborate on a student answer, finish a student's thought, rephrase and modify mathematical language, validate a response, synthesize input from multiple students
Explain	Explain idea or concept, distinguish concepts, clarify method, illustrate with an example, contrast problem structures
Model reasoning	Justify a solution or algorithm, correct flawed student work, provide solution component, provide alternate solution
Norms & expectations	Explicate norms such as mathematical conventions, notation, terminology, expectations for justifications, or the intended form of an answer

An example illustrates two of these purposes. During one group discussion students were trying to understand the range of a set of shoe sizes (that varied from size 6 to 12) in a shoe store by counting the available sizes. They had some trouble deciding whether the range was six or seven (by whether they counted the smallest size), as well as whether one would need to count half sizes. To avoid this quandary one student joked that her store just wouldn't offer half sizes! The teacher decided to disclose the following explanation to help clarify range:

Teacher: You don't offer half sizes!? So, okay, so it's like—So, it sounds like, okay, it sounds like these are two different numbers. When you're talking about the range, um, that's saying I go—I, like, I encompass this amount. Whereas, if I say I offer fourteen sizes, that's a count of the number of sizes you offer. And I think those are two different numbers. (*Disclose-Explain*)

In something of a tentative follow-up, one student ventured her own definition of range, included below. The teacher amplified this student's input:

Student: Is the range the one number that's how many numbers are in between the smallest and the greatest?

Teacher: Yeah, it's the distance between the smallest and the greatest. So, that's a six. (*Disclose-Amplify*)

The teacher amplified the student's input by strategically modifying the mathematical imagery and accompanying language. Instead of the problematic phrase "how many numbers" (there are actually infinitely many numbers between any two real numbers), she used the more precise term "distance." In addition, she modeled appropriate usage of her definition to provide the answer to this particular example—the range of shoe sizes varying between 6 and 12 is six.

Validate

The seventh and final type of mathematical telling is to validate a student expression of mathematics as correct or incorrect. Validation was unique in that it could occur as the only function

of a telling utterance or in combination with one of the other six telling functions (for example, many disclose–amplify utterances also validated student input). Validation was the least reliable code to discern in the data. At times it was ambiguous due to words such as “yeah, okay, uh-huh” and “alright” that were also social conversational markers. Other times validation occurred in the discourse simply by repeating student language with positive emphasis or negative skepticism. Other implicit interactional norms or gestures may have served as validation and gone unnoticed. I conservatively marked discourse as validate only when the speech before and after the incident in question showed evidence that the teacher and students treated it as validation. Even so, validation was a frequent type of telling, occurring in well over half of the mathematical conversations the teacher held with groups and individuals.

Discussion

Three important findings across the seven telling types distinguish this study from prior work in inquiry. First was to recognize the frequently overlooked mathematical contributions in subtle acts such as assessing and interpreting student thought. Second was to make mathematical contributions of indirect mathematical aid, such as guidance, more transparent. Third was to acknowledge that overt forms of telling, such as disclosing items and validating student work, were pervasive and integrated components of the teacher’s inquiry-based instruction.

Conclusion

The primary contribution of this study is to empirically expand the construct of telling as it occurs in contemporary pedagogical spaces such as inquiry. Telling is more than “simply telling students whether their answers are right or wrong or giving students correct answers” (Chazan & Ball, 1999, p. 2). Mathematical telling comprises a rich vocabulary for talking about some of the mathematical content and pedagogical functions of teacher discourse. Telling as a conceptual space was enriched to include less conspicuous forms of telling whose embedded mathematical structures have frequently been eclipsed by the glaring omission of more conspicuous mathematical information.

Inquiry has informally been set apart from more direct modes of teaching by its pervasive lack of “telling.” The descriptive case study presented here suggests that this need not be the case. The examples of mathematical telling presented in the framework were a consistent and integrated component of the teacher’s inquiry-based practice. Naturally, not every instance of mathematical telling was ideal, and the study should not be taken to justify the indiscriminate use of telling. However, mathematical telling as a practice is not automatically harmful, unwarranted, or inferior to less mathematically saturated discourse simply by virtue of initiating mathematical ideas (see Lobato et al., 2005).

The main implication of these considerations is the need to take a more nuanced and contextual orientation to telling. Mathematical telling practices should be viewed as one set of many available instructional tools for creating structure and managing student activity, along with mathematical tasks, tools, physical environments, and collaborative interactional norms.

Further research should investigate teacher awareness of and intentional use of telling practices, the influence of mathematical telling on student thinking, the use of telling practices in whole-class discussions and other instructional settings, and the situated relationship of mathematical telling to the other parts of the teaching environment as a whole.

References

- Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal*, 93, 373-397.
- Baxter, J. A., & Williams, S. R. (2010). Social and analytic scaffolding in middle school mathematics: Managing the dilemma of telling. *Journal of Mathematics Teacher Education*, 13, 7-26.

- Chazan, D., & Ball, D. L. (1999). Beyond being told not to tell. *For the Learning of Mathematics*, 19(2), 2-10.
- Chiu, M. M. (2004). Adapting teacher interventions to student needs during cooperative learning: How to improve student problem solving and time on-task. *American Educational Research Journal*, 41, 365-399.
- Clarke, D. (1997). The changing role of the mathematics teacher. *Journal for Research in Mathematics Education*, 28, 278-308.
- Cohen, E. G. (1994). Restructuring the classroom: Conditions for productive small groups. *Review of Educational Research*, 64, 1-35.
- Dekker, R., & Elshout-Mohr, M. (2004). Teacher interventions aimed at mathematical level raising during collaborative learning. *Educational Studies in Mathematics*, 56, 39-65.
- Ding, M., Li, X., Piccolo, D., & Kulm, G. (2007). Teacher interventions in cooperative-learning mathematics classes. *The Journal of Educational Research*, 100, 162-175.
- Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 28, 524-549.
- Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27, 29-63.
- Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education*, 36, 101-136.
- Lubienski, S. T. (2002). Research, reform, and equity in U.S. mathematics education. *Mathematical Thinking and Learning*, 4, 103-125.
- Rittenhouse, P. S. (1998). The teacher's role in mathematical conversations: Stepping in and stepping out. In M. Lampert & M. L. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning* (pp. 163-189). Cambridge, United Kingdom: Cambridge University Press.
- Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26, 114-145.
- Simon, M. A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning*, 6, 91-104.
- Smith, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education*, 27, 387-402.
- Staples, M. (2007). Supporting whole-class collaborative inquiry in a secondary mathematics classroom. *Cognition and Instruction*, 25, 161-217.
- Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, 10, 313-340.
- Tyminski, A. M. (2010). Teacher lust: Reconstructing the construct for mathematics education. *Journal of Mathematics Teacher Education*, 13, 295-311.
- von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning*. Bristol, PA: The Falmer Press.
- Webb, N. M. (2009). The teacher's role in promoting collaborative dialogue in the classroom. *British Journal of Educational Psychology*, 79, 1-28.
- Webb, N. M., Franke, M. L., De, T., Chan, A. G., Freund, D., Shein, P., & Melkonian, D. K. (2009). 'Explain to your partner': Teachers' instructional practices and students' dialogue in small groups. *Cambridge Journal of Education*, 39, 49-70.
- Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpiska (Eds.), *Language and communication in the mathematics classroom* (pp. 167-178). Reston, VA: National Council of Teachers of Mathematics.